

SURDS AND INDICES

(1)

$$\sqrt{19-4\sqrt{x}} = \sqrt{12} - \sqrt{7}$$

squaring on both the sides

$$(\sqrt{19-4\sqrt{x}})^2 = (\sqrt{12} - \sqrt{7})^2$$

$$19 - 4\sqrt{x} = 12 + 7 - 2\sqrt{12} \cdot \sqrt{7}$$

$$19 - 4\sqrt{x} = 19 - 2\sqrt{84}$$

$$-4\sqrt{x} = -2\sqrt{84}$$

$$2\sqrt{x} = \sqrt{84}$$

$$(2\sqrt{x})^2 = (\sqrt{84})^2$$

$$4x = 84$$

$$x = \frac{84}{4} = 21$$

$$x = 21$$

option (3)

$$Q2 \text{ let } \sqrt{6+x} = x$$

$$\sqrt{6+x} = x$$

$$(\sqrt{6+x})^2 = (x)^2$$

$$6+x = x^2$$

$$x^2 - x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x+2)(x-3) = 0$$

$$x+2=0 \text{ OR } x-3=0$$

$$x = -2 \text{ OR } x = 3$$

↓ Discarded

$$x = 3$$

Q3

$$x = 8 - \sqrt{60}$$

$$\sqrt{x} = \sqrt{8 - \sqrt{60}}$$

$$= \sqrt{8 - \sqrt{4 \times 15}}$$

$$\sqrt{x} = \sqrt{8 - 2\sqrt{15}}$$

$$= \sqrt{(\sqrt{5} - \sqrt{3})^2}$$

$$\sqrt{x} = \sqrt{5} - \sqrt{3}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{\sqrt{5} + \sqrt{3}}{5 - 3} = \frac{\sqrt{5} + \sqrt{3}}{2}$$

option (1)

$$\frac{1}{2} \left[\sqrt{x} + \frac{2}{\sqrt{x}} \right] = \frac{1}{2} \left[\sqrt{5} - \sqrt{3} + \frac{2}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \right]$$

$$= \frac{1}{2} \left[\sqrt{5} - \sqrt{3} + \sqrt{5} + \sqrt{3} \right]$$

$$= \frac{1}{2} [2\sqrt{5}]$$

$$= \sqrt{5}$$

Q.4

$$\sqrt{50} + \sqrt{48} = k(\sqrt{3} + \sqrt{2})$$

$$(\sqrt{50} + \sqrt{48})^2 = [k(\sqrt{3} + \sqrt{2})]^2$$

$$\sqrt{50} + \sqrt{48} = k^2(3+2+2\sqrt{6}) = k^2(5+2\sqrt{6})$$

$$(\sqrt{50} + \sqrt{48})^2 = [k^2(5+2\sqrt{6})]^2 = k^4(5+2\sqrt{6})^2$$

$$50 + 48 + 2\sqrt{50} \times \sqrt{48} = k^4(25 + 24 + 20\sqrt{6})$$

$$98 + 2 \times 5\sqrt{2} \times 4\sqrt{3} = k^4(49 + 20\sqrt{6})$$

$$98 + 2 \times 20\sqrt{6} = k^4(49 + 20\sqrt{6})$$

$$2(49 + 20\sqrt{6}) = k^4$$

$$(49 + 20\sqrt{6})$$

$$\therefore k^4 = 2, \quad k = 2^{\frac{1}{4}}$$

option (3)

Q.5

$$\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a+b\sqrt{5}$$

$$\frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}} = a+b\sqrt{5}$$

$$\frac{(4+3\sqrt{5})^2}{16-45} = a+b\sqrt{5}$$

$$\frac{16+45+2 \times 4 \times 3\sqrt{5}}{-29} = a+b\sqrt{5}$$

$$-29$$

$$\frac{61+24\sqrt{5}}{-29} = a+b\sqrt{5}$$

$$-29 - \frac{61}{29} + \left(\frac{24}{29}\right)\sqrt{5} = a+b\sqrt{5}$$

$$a = -\frac{61}{29}, \quad b = -\frac{24}{29}$$

option (4)

Q.6

$$x = \sqrt{5}, y = \sqrt[4]{10}, z = \sqrt[3]{6}$$

$$\text{L.C.M.}(2, 4, 3) = 12$$

$$x = \sqrt[12]{5^6}, y = \sqrt[12]{10^3}, z = \sqrt[12]{6^4}$$

$$\Rightarrow x > z > y \quad (1) \quad y < z < x \quad \text{option (2)}$$

Q.7

$$x = \sqrt[3]{9}, y = \sqrt[4]{11}, z = \sqrt[6]{17}$$

$$\text{L.C.M.}(3, 4, 6) = 12$$

$$x = \sqrt[12]{9^4}, y = \sqrt[12]{11^3}, z = \sqrt[12]{17^2}$$

$$x > y > z \quad \text{option (1)}$$

Q.8

$$\sqrt[3]{4}, \sqrt[4]{5}, \sqrt[4]{6}, \sqrt[3]{8}$$

$$\text{L.C.M.}(3, 4, 4, 3) = 12$$

$$\sqrt[12]{4^8}, \sqrt[12]{5^3}, \sqrt[12]{6^3}, \sqrt[12]{8^4}$$

$$\sqrt[12]{256}, \sqrt[12]{125}, \sqrt[12]{216}, \sqrt[12]{4096}$$

Smallest

$$\sqrt[4]{5}$$

option

(2)

Q.9

$$\frac{1}{\sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}} = \frac{\sqrt{10} - \sqrt{14} - \sqrt{15} + \sqrt{21}}{K}$$

$$K = (\sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21})(\sqrt{10} - \sqrt{14} - \sqrt{15} + \sqrt{21})$$

$$K = \{(\sqrt{10} + \sqrt{14}) + (\sqrt{14} + \sqrt{15})\} \{(\sqrt{10} + \sqrt{21}) - (\sqrt{14} + \sqrt{15})\}$$

$$K = (\sqrt{10} + \sqrt{21})^2 - (\sqrt{14} + \sqrt{15})^2$$

$$kl = (10 + 21 + 2\sqrt{210}) - (4 + 15 + 2\sqrt{210})$$

$$= 31 + 2\sqrt{210} - 29 - 2\sqrt{210}$$

$$kl = 2, \quad d = 2 | k$$

option (3)

(10)

$$\sqrt{11 + \sqrt{112}} = \sqrt{11 + \sqrt{2 \times 2 \times 28}}$$

$$= \sqrt{11 + 2\sqrt{28}} = \sqrt{(\sqrt{7} + \sqrt{4})^2}$$

$$= \sqrt{7} + \sqrt{4} = \sqrt{7} + 2$$

$$\sqrt{11 + \sqrt{112}} = \sqrt{7} + 2$$

option (1)

(11)

$$x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$$

$$x - 2 = 2^{\frac{1}{3}} + 2^{\frac{2}{3}} \quad \text{--- (1)}$$

$$(x-2)^3 = (2^{\frac{1}{3}} + 2^{\frac{2}{3}})^3 \quad \left[\begin{array}{l} (a+b)^3 = a^3 + b^3 + 3ab(a+b) \\ (a-b)^3 = a^3 - b^3 - 3ab(a-b) \end{array} \right]$$

$$x^3 - 2^3 - 3x \times 2(x-2) = (2^{\frac{1}{3}})^3 + (2^{\frac{2}{3}})^3 + 3 \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{2}{3}} (2^{\frac{1}{3}} + 2^{\frac{2}{3}})$$

$$x^3 - 8 - 6x(x-2) = 2 + 2^2 + 3 \cdot 2^1 (x-2) \quad (\text{from 1})$$

$$x^3 - 8 - 6x^2 + 12x = 2 + 4 + 6(x-2)$$

$$x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$$

$$x^3 - 6x^2 + 6x = -12 + 14 = 2$$

$$x^3 - 6x^2 + 6x - 12 = 2$$

Ans

option (D)

$$(12) \quad \sqrt{5+2\sqrt{6}} = \sqrt{(\sqrt{3}+\sqrt{2})^2} = \sqrt{3}+\sqrt{2}$$

option (4)

$$(13) \quad \sqrt{21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}}$$

$$= \sqrt{21-2 \times 2\sqrt{5}+2 \times 4\sqrt{3}-2 \times 2\sqrt{15}}$$

$$= \sqrt{21-2\sqrt{20}+2\sqrt{48}-2\sqrt{60}}$$

$$= \sqrt{(-\sqrt{5}+\sqrt{4}+\sqrt{12})^2} = -\sqrt{5}+\sqrt{4}+\sqrt{12}$$

option (6)

$$(14) \quad \frac{1}{\sqrt{12+\sqrt{140}}} + \frac{1}{\sqrt{8-\sqrt{60}}}$$

$$\sqrt{12+\sqrt{140}} = \sqrt{12+\sqrt{2 \times 2 \times 35}} = \sqrt{12+2\sqrt{35}}$$

$$= \sqrt{(\sqrt{7}+\sqrt{5})^2} = \sqrt{7}+\sqrt{5}$$

$$\sqrt{12+\sqrt{140}} = \sqrt{7}+\sqrt{5}$$

$$\sqrt{8-\sqrt{60}} = \sqrt{8-\sqrt{2 \times 2 \times 15}} = \sqrt{8-2\sqrt{15}}$$

$$= \sqrt{(\sqrt{5}-\sqrt{3})^2} = \sqrt{5}-\sqrt{3}$$

$$= \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{1}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}+\sqrt{3}}{5-3}$$

$$\frac{\sqrt{7}-\sqrt{3}}{2} + \frac{\sqrt{5}+\sqrt{2}}{2}$$

$$\frac{\sqrt{7}-\sqrt{3}+\sqrt{5}+\sqrt{2}}{2} = \frac{\sqrt{7}+\sqrt{3}}{2}$$

Rationalize Numerator

$$\begin{aligned} \frac{\sqrt{7}-\sqrt{3}}{2} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} &= \frac{(\sqrt{7})^2 - (\sqrt{3})^2}{\sqrt{7}+\sqrt{3}} \\ &= \frac{7-3}{2(\sqrt{7}+\sqrt{3})} = \frac{4}{2(\sqrt{7}+\sqrt{3})} \\ &= \frac{2}{\sqrt{(\sqrt{7}+\sqrt{3})^2}} = \frac{2}{\sqrt{10+2\sqrt{21}}} \\ &= \frac{2}{\sqrt{10-2\sqrt{21}}} \quad \text{option (W)} \end{aligned}$$

Q.18

$$\sqrt{23+x\sqrt{10}} = \sqrt{18} + \sqrt{5}$$

$$(\sqrt{23+x\sqrt{10}})^2 = (\sqrt{18} + \sqrt{5})^2$$

$$23+x\sqrt{10} = 18+5+2\sqrt{90}$$

$$23+x\sqrt{10} = 23+2\sqrt{90}$$

$$x\sqrt{10} = 2\sqrt{90}, \quad x = \frac{2\sqrt{90}}{\sqrt{10}} = \frac{2 \times \sqrt{9} \times \sqrt{10}}{\sqrt{10}}$$

$$\boxed{x = 2\sqrt{9}} = x = 2 \times 3 = 6$$

$$\boxed{x=6} \quad \text{option (T)}$$

$$\textcircled{16} \quad \frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = a + \sqrt{b} \quad \text{then } (a, b)$$

$$\frac{4+3\sqrt{3}}{\sqrt{7+2\sqrt{12}}} = a + \sqrt{b}$$

$$\frac{4+3\sqrt{3}}{\sqrt{(2+\sqrt{3})^2}} = a + \sqrt{b}$$

$$\frac{4+3\sqrt{3}}{2+\sqrt{3}} = a + \sqrt{b}$$

$$\frac{4+3\sqrt{3}}{2+\sqrt{3}} = a + \sqrt{b}$$

$$\frac{4+3\sqrt{3}}{(2+\sqrt{3})} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = a + \sqrt{b}$$

$$\frac{8 - 4\sqrt{3} + 6\sqrt{3} - 9}{4-3} = a + \sqrt{b}$$

$$-1 + 2\sqrt{3} = a + \sqrt{b}$$

$$-1 + \sqrt{12} = a + \sqrt{b}$$

$$a = -1, b = 12$$

$$(a, b) = (-1, 12)$$

option ①

$$(17) \quad x = 3 - \sqrt{5},$$

$$\sqrt{x} = \sqrt{3 - \sqrt{5}}$$

$$= \sqrt{\frac{2}{3}(3 - \sqrt{5})} = \sqrt{\frac{6 - 2\sqrt{5}}{2}}$$

$$= \frac{1}{\sqrt{2}} \sqrt{6 - 2\sqrt{5}} = \frac{1}{\sqrt{2}} \sqrt{(\sqrt{5} - \sqrt{1})^2}$$

$$\sqrt{x} = \frac{1}{\sqrt{2}} (\sqrt{5} - \sqrt{1}) = \frac{1}{\sqrt{2}} (\sqrt{5} - 1)$$

$$\sqrt{3x - 2} = \sqrt{3(3 - \sqrt{5}) - 2} = \sqrt{9 - 3\sqrt{5} - 2} = \sqrt{7 - 3\sqrt{5}}$$

$$\sqrt{3x - 2} = \sqrt{7 - \sqrt{45}} = \sqrt{\frac{2}{2} (7 - \sqrt{45})}$$

$$\sqrt{3x - 2} = \sqrt{\frac{14 - 2\sqrt{45}}{2}} = \frac{\sqrt{14 - 2\sqrt{45}}}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left(\sqrt{(\sqrt{9} - \sqrt{5})^2} \right)$$

$$\sqrt{3x - 2} = \frac{1}{\sqrt{2}} (\sqrt{9} - \sqrt{5}) = \frac{\sqrt{9} - \sqrt{5}}{\sqrt{2}}$$

$$\frac{\sqrt{x}}{\sqrt{2} + \sqrt{3x - 2}} = \frac{\frac{\sqrt{5} - 1}{\sqrt{2}}}{\sqrt{2} + \frac{\sqrt{9} - \sqrt{5}}{\sqrt{2}}} = \frac{\sqrt{5} - 1}{\sqrt{9} - \sqrt{5}}$$

$$= \frac{\sqrt{5} - 1}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}}$$

$$= \frac{3\sqrt{5} + 5 - 3 - \sqrt{5}}{9 - 5}$$

$$= \frac{\cancel{2\sqrt{5}} + 2}{4}$$

$$= \frac{\sqrt{5}-1}{\sqrt{2}} = \frac{\sqrt{5}-1}{2+3-\sqrt{5}} = \frac{\sqrt{5}-1}{5-\sqrt{5}} \times \frac{5+\sqrt{5}}{5+\sqrt{5}}$$

$$= \frac{\sqrt{2}}{5\sqrt{5} + 5\sqrt{5} - 5 - \sqrt{5}} = \frac{\sqrt{2}}{20\sqrt{5} - 5} = \frac{1}{\sqrt{5}}$$

$$\frac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - 2} = \frac{1}{\sqrt{5}} \quad \text{Ans}$$

option ①

$$\textcircled{18} \quad \frac{a + \sqrt{a^2 - 1}}{a - \sqrt{a^2 - 1}} \times \frac{a + \sqrt{a^2 - 1}}{a + \sqrt{a^2 - 1}}$$

$$= \frac{(a + \sqrt{a^2 - 1})^2}{a^2 - (a^2 - 1)} = \frac{a^2 + 2a\sqrt{a^2 - 1} + a^2 - 1}{a^2 - a^2 + 1}$$

$$= 2a^2 + 2a\sqrt{a^2 - 1} - 1$$

option ①

$$\textcircled{19} \quad \frac{1}{2-\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{4}{3-\sqrt{5}}$$

$$\frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{4}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{2+\sqrt{3}}{4-3} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{4(3+\sqrt{5})}{9-5}$$

$$= \frac{2+\sqrt{3}}{1} + (\sqrt{3}-\sqrt{2}) + \cancel{4} \frac{3+\sqrt{5}}{\cancel{4}}$$

$$= 2-\sqrt{3}-\sqrt{3}+\sqrt{2}+3+\sqrt{5}$$

$$= 5+\sqrt{2}+\sqrt{5}$$

option (2)

(20)

$$\frac{4}{2+\sqrt{3}+\sqrt{7}} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\frac{4}{(2+\sqrt{3})+\sqrt{7}} \times \frac{(2+\sqrt{3})-(\sqrt{7})}{(2+\sqrt{3})-(\sqrt{7})} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\frac{4(2+\sqrt{3}-\sqrt{7})}{(2+\sqrt{3})^2 - (\sqrt{7})^2} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\cancel{8+4\sqrt{3}} \frac{4(2+\sqrt{3}-\sqrt{7})}{4+3+4\sqrt{3}-7} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\frac{4(2+\sqrt{3}-\sqrt{7})}{4+4\sqrt{3}-7} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\cancel{4} \frac{(2+\sqrt{3}-\sqrt{7})}{4\sqrt{3}} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\frac{2+\sqrt{3}-\sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$(2 + \sqrt{3} - \sqrt{7})\sqrt{3} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\frac{2\sqrt{3} + \sqrt{9} - \sqrt{21}}{3} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\sqrt{\frac{4 \times 3}{9}} + \sqrt{\frac{9}{9}} - \sqrt{\frac{21}{9}} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$\sqrt{\frac{4}{3}} + 1 - \sqrt{\frac{7}{3}} = \sqrt{a} + \sqrt{b} + \sqrt{c}$$

$$a=1, \quad b=\frac{4}{3}, \quad c=\frac{21}{9}$$

option. (1)

$$(21) \quad x = 5 - 2\sqrt{6}$$

$$\sqrt{x} = \sqrt{5 - 2\sqrt{6}} = \sqrt{(\sqrt{3} - \sqrt{2})^2} = \sqrt{3} - \sqrt{2}$$

$$\sqrt{\frac{x}{2}} + \frac{1}{\sqrt{2x}} = \frac{\sqrt{x}}{\sqrt{2}} + \frac{1}{\sqrt{2x}}$$

$$= \frac{1}{\sqrt{2}} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)$$

$$= \frac{1}{\sqrt{2}} (\sqrt{3} - \sqrt{2} + (\sqrt{3} + \sqrt{2}))$$

$$= \frac{1}{\sqrt{2}} (\cancel{\sqrt{2}} + \sqrt{2} - \cancel{\sqrt{3}} + \sqrt{3})$$

$$= \frac{1}{\sqrt{2}} \times 2\sqrt{2} = 2$$

option. (2)

$$(22) \quad \frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2}$$

$$= \frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2}$$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{6-3} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{6-2} + \frac{2\sqrt{3}(\sqrt{6}-2)}{6-4}$$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{3} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{4} + \frac{2\sqrt{3}(\sqrt{6}-2)}{2}$$

$$= \sqrt{12} + \sqrt{6} - \sqrt{18} - \sqrt{6} + \sqrt{18} - 2\sqrt{3}$$

$$= 2\sqrt{3} - 2\sqrt{3} = 0$$

option. (3)

$$(23) \quad \frac{3+\sqrt{6}}{\sqrt{75}-\sqrt{48}-\sqrt{32}+\sqrt{50}}$$

$$= \frac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}}$$

$$= \frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{3}+3\sqrt{2}+\sqrt{18}+\sqrt{12}}{3-2}$$

$$= 3\sqrt{3}+3\sqrt{2}+3\sqrt{2}+2\sqrt{3}$$

$$\sqrt{3} + \sqrt{2}$$

$$= \sqrt{3} + \sqrt{2}$$

option (4)

(24)

$$\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}} = a + b\sqrt{3}$$

$$\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}} \times \frac{33+19\sqrt{3}}{33+19\sqrt{3}} = a + b\sqrt{3}$$

$$\therefore \sqrt{\frac{198+114\sqrt{3}+66\sqrt{3}+114}{1089-1083}} = a + b\sqrt{3}$$

$$\therefore \sqrt{\frac{312+180\sqrt{3}}{6}} = a + b\sqrt{3}$$

$$\sqrt{6(52+30\sqrt{3})} = a + b\sqrt{3}$$

$$\sqrt{52+2 \times 15\sqrt{3}} = a + b\sqrt{3}$$

$$\sqrt{52+2 \times 225 \times 3} = a + b\sqrt{3}$$

$$\sqrt{52+2 \times 675} = a + b\sqrt{3}$$

$$\therefore \sqrt{52+1350} = a + b\sqrt{3}$$

$$\sqrt{1402} = a + b\sqrt{3}$$

$$5 + 3\sqrt{3} = a + b\sqrt{3}$$

$$a = 5, b = 3$$

$$a + b = 5 + 3 = 8$$

(25)

$$\frac{\left(p + \frac{1}{q}\right)^p \left(b - \frac{1}{q}\right)^q}{\left(q + \frac{1}{p}\right)^q \left(q - \frac{1}{p}\right)^q} = \left(\frac{p}{q}\right)^x$$

$$\left(\frac{p + \frac{1}{q}}{q + \frac{1}{p}}\right)^p \left(\frac{b - \frac{1}{q}}{q - \frac{1}{p}}\right)^q = \left(\frac{p}{q}\right)^x$$

$$\left(\frac{\frac{pq+1}{q}}{\frac{pq+1}{p}}\right)^p \left(\frac{\frac{pq-1}{q}}{\frac{pq-1}{p}}\right)^q = \left(\frac{p}{q}\right)^x$$

$$\left(\frac{\cancel{pq+1} \times \frac{p}{pq+1}}{q}\right)^p \left(\frac{\cancel{pq-1} \times \frac{p}{pq-1}}{q}\right)^q = \left(\frac{p}{q}\right)^x$$

$$\left(\frac{p}{q}\right)^p \left(\frac{p}{q}\right)^q = \left(\frac{p}{q}\right)^x$$

$$\left(\frac{p}{q}\right)^{p+q} = \left(\frac{p}{q}\right)^x$$

$$x = p + q$$

o/p. 25

(26)
$$\frac{5+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{10+5\sqrt{3}+2\sqrt{3}+3}{4+2\sqrt{3}-2\sqrt{3}-3} = \frac{13+7\sqrt{3}}{1} = 13+7\sqrt{3}$$

option ①

(27)
$$\sqrt{10+2\sqrt{21}} = \sqrt{7} + \sqrt{3}$$

$$\sqrt{(\sqrt{7}+\sqrt{3})^2} = \sqrt{7} + \sqrt{3}$$

$$\sqrt{7} + \sqrt{3} = \sqrt{7} + \sqrt{3}$$

$$\sqrt{3} = \sqrt{21}, \quad \text{option ②}$$

(28)
$$x = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$$

$$x = \sqrt{\frac{7+2\sqrt{12}}{7-2\sqrt{12}}} = \sqrt{\frac{(\sqrt{4}+\sqrt{3})^2}{(\sqrt{4}-\sqrt{3})^2}} = \frac{\sqrt{4}+\sqrt{3}}{\sqrt{4}-\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})^2}{(2)^2 - (\sqrt{3})^2}$$

$$x = \frac{4+4\sqrt{3}+3}{1} = 7+4\sqrt{3}$$

$$\frac{1}{x} = 7 - 4\sqrt{3}$$

$$x + \frac{1}{x} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$$

$$\boxed{x + \frac{1}{x} = 14}$$

option (3)

(29)

$$3 + \sqrt{27} + \sqrt{75} = a + b\sqrt{3}$$

$$3 + 3\sqrt{3} + 5\sqrt{3} = a + b\sqrt{3}$$

$$3 + 8\sqrt{3} = a + b\sqrt{3}$$

$$a = 3, b = 8, (a, b) = (3, 8)$$

option (2)

(30)

$$\sqrt[3]{64 - 9} = 3$$

$$(\sqrt[3]{64 - 9})^3 = 3^3$$

$$64 - 9 = 27, 64 = 27 + 9 = 36$$

$$64 = 36, \gamma = \frac{36}{6} = 6$$

$$\boxed{7 - 6}$$

option (3)

(31)

$$a = \frac{1}{3 - 2\sqrt{2}}, b = \frac{1}{3 + 2\sqrt{2}}$$

$$a = \frac{1}{3 - 2\sqrt{2}} \times \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}}$$

$$= \frac{3 + 2\sqrt{2}}{9 - 8} = 3 + 2\sqrt{2}$$

$$b = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = \frac{3-2\sqrt{2}}{1} = 3-2\sqrt{2}$$

$$b = 3-2\sqrt{2}$$

$$a+b = 3+2\sqrt{2} + 3-2\sqrt{2} = 6$$

$$ab = (3+2\sqrt{2})(3-2\sqrt{2}) = 3^2 - (2\sqrt{2})^2 = 9-8=1$$

$$a^2b + ab^2 = ab(a+b) = 1 \cdot 6 = 6$$

$$\boxed{a^2b + ab^2 = 6} \quad \text{Ans}$$

option ②

$$\textcircled{32} \quad x = 7+4\sqrt{3}, \quad xy = 1$$

$$\frac{1}{x} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{49-48} = \frac{7-4\sqrt{3}}{1} = 7-4\sqrt{3}$$

$$xy = 1, \quad \cancel{y = \frac{1}{x}} \quad x = \frac{1}{y}, \quad \frac{1}{y} = x, \quad \frac{1}{y} = 7+4\sqrt{3}$$

$$\frac{1}{x} + \frac{1}{y} = 7-4\sqrt{3} + 7+4\sqrt{3} = 14$$

$$\left(\frac{1}{x} + \frac{1}{y}\right)^2 = \frac{1}{x^2} + \frac{1}{y^2} + 2 \cdot \frac{1}{x} \cdot \frac{1}{y}$$

$$(14)^2 = \frac{1}{x^2} + \frac{1}{y^2} + \frac{2}{1}$$

$$196 - 2 = \frac{1}{x^2} + \frac{1}{y^2}$$

$$\boxed{\frac{1}{x^2} + \frac{1}{y^2} = 194} \quad \text{Ans} \quad \text{option ①}$$

(33) $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

We know that

$$x^3 + y^3 + z^3 = 3xyz \quad \text{if } x + y + z = 0$$

put $x = \sqrt[3]{a}$, $y = \sqrt[3]{b}$, $z = \sqrt[3]{c}$

$$x + y + z = \sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$$

$$\therefore (\sqrt[3]{a})^3 + (\sqrt[3]{b})^3 + (\sqrt[3]{c})^3 = 3 \sqrt[3]{a} \cdot \sqrt[3]{b} \cdot \sqrt[3]{c}$$

$$a + b + c = 3 \sqrt[3]{abc}$$

$$(a + b + c)^2 = (3 \sqrt[3]{abc})^2 = 27 abc$$

$$(a + b + c)^2 = 27 abc \quad \text{Ans}$$

(34) $x = 5 + 2\sqrt{6}$

$$\sqrt{x} = \sqrt{5 + 2\sqrt{6}} = \sqrt{(\sqrt{3} + \sqrt{2})^2} = \sqrt{3} + \sqrt{2}$$

$$\frac{x+1}{\sqrt{x}} = \frac{5+2\sqrt{6}+1}{\sqrt{3}+\sqrt{2}} = \frac{4+2\sqrt{6}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{4\sqrt{3} - 4\sqrt{2} + 2\sqrt{18} - 2\sqrt{2}}{3-2}$$

$$= \frac{4\sqrt{3} - 4\sqrt{2} - 4\sqrt{2} + 6\sqrt{2}}{1}$$

$$= 4\sqrt{3} - 4\sqrt{2} - 4\sqrt{2} + 6\sqrt{2} = 2\sqrt{2}$$

Option (D)

(35)

$$\frac{(x^{a+b})^2 \cdot (x^{b+c})^2 \cdot (x^{c+a})^2}{(x^a \cdot x^b \cdot x^c)^4}$$

$$= \frac{x^{2(a+b)} \cdot x^{2(b+c)} \cdot x^{2(c+a)}}{x^{4(a+b+c)}} = \frac{x^{2a+2b+2c}}{x^{4a+4b+4c}} = 1$$

option (1)

$$(36) \text{ let } 4^x = 5^y = 20^z = k$$

$$4^x = k, \quad 4 = k^{1/x}$$

$$20^z = k$$

$$5^y = k, \quad 5 = k^{1/y}$$

$$20 = k^{1/z}$$

$$20 = k^{1/x + 1/y} \Rightarrow k^{1/z} = k^{1/x + 1/y}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z} \Rightarrow \frac{1}{x} + \frac{1}{y} - \frac{1}{z} = 0$$

$$\frac{x+y}{xy} = \frac{1}{z} \Rightarrow z = \frac{xy}{x+y}$$

option (4)

(37)

$$\frac{a^3}{x} \cdot \frac{b^3}{x} \cdot \frac{3ab(a+b)}{x} = (2^5)^{25}, \quad a+b=5$$

$$\frac{a^3 + b^3 + 3ab(a+b)}{x} = (2^5)^{25}$$

$$\frac{x^{a^3+b^3+3ab(a+b)}}{x^{5^3}} = 2^{125} \Rightarrow x^{125} = 2^{125}$$

$$(x=2)$$

option (1)

(38)

$$\begin{aligned}
 & \left(\frac{1}{x^{a-b}} \right)^{\frac{1}{c-a}} \left(\frac{1}{x^{b-c}} \right)^{\frac{1}{a-b}} \left(\frac{1}{x^{c-a}} \right)^{\frac{1}{b-c}} \\
 &= x^{\frac{1}{(a-b)(c-a)}} \cdot x^{\frac{1}{(b-c)(a-b)}} \cdot x^{\frac{1}{(c-a)(b-c)}} \\
 &= x^{\frac{1}{(a-b)(c-a)} + \frac{1}{(b-c)(a-b)} + \frac{1}{(c-a)(b-c)}} \\
 &= x^{\frac{b-c + c-a + a-b}{(a-b)(b-c)(c-a)}} \\
 &= x^0 = 1
 \end{aligned}$$

option (D)

(39)

$$\begin{aligned}
 & \frac{(x^{2^{n-1}} + y^{2^{n-1}})(x^{2^{n-1}} - y^{2^{n-1}})}{x^{2^n} - y^{2^n}} \\
 &= \frac{(x^{2^{n-1}})^2 - (y^{2^{n-1}})^2}{x^{2^n} - y^{2^n}} \\
 &= \frac{x^{2! \cdot 2^{n-1}} - y^{2! \cdot 2^{n-1}}}{x^{2^n} - y^{2^n}} = \frac{x^{2^n - 1} - y^{2^n - 1}}{x^{2^n} - y^{2^n}}
 \end{aligned}$$

$$= \frac{x^{2n} - y^{2n}}{x^{2n} - y^{2n}} = 1$$

option ①

$$\begin{aligned} \textcircled{40} & \left(\left((625)^{-1/2} \right)^{-1/4} \right)^2 \\ &= (625)^{\frac{2}{84}} = (625)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5 \end{aligned}$$

option ②

$$\begin{aligned} \textcircled{41} & \left(5 (8^{1/3} + 27^{1/3})^3 \right)^{1/4} \\ &= \left(5 (5)^3 \right)^{1/4} = (5^4)^{1/4} = 5^1 = 5 \end{aligned}$$

option ③

$$\begin{aligned} \textcircled{42} & (0.000729)^{-3/4} \times (0.09)^{-3/4} \\ &= \left(\frac{729}{10^6} \right)^{-3/4} \times \left(\frac{9}{10^2} \right)^{-3/4} \\ &= \left(\frac{10^6}{729} \right)^{3/4} \times \left(\frac{10^2}{3^2} \right)^{3/4} \\ &= \frac{(10^6)^{3/4}}{(9^3)^{3/4}} \times \frac{(10^2)^{3/4}}{(3^2)^{3/4}} = \frac{10^{9/2} \cdot 10^{3/2}}{9^{9/4} \cdot 3^{6/4}} \end{aligned}$$

$$= \frac{10^{\frac{12}{2}}}{3^{\frac{18}{4}} \cdot 3^{\frac{6}{4}}} = \frac{10^6}{3^{\frac{24}{4}} \cdot 3^{\frac{6}{4}}} = \frac{10^6}{3^6}$$

$$= \frac{10^6}{3^6} \quad \text{option (4)}$$

(43)

$$800 = 8 \times 10^8 \times x^{-3/2}$$

$$\frac{800}{8 \times 10^8} = x^{-3/2}$$

$$10^{-6} = x^{-3/2}$$

$$\therefore 10^{-6} = x^{-3/2}$$

$$\frac{1}{10^6} = \frac{1}{x^{3/2}} \Rightarrow x^{3/2} = 10^6$$

$$x = (10^6)^{2/3} = 10^4$$

$$x = 10^4$$

option (3)

(44)

$$\left(-\frac{5}{6}\right)^{3/4} \div \left(-\frac{5}{6}\right)^{7/6} = \left(-\frac{5}{6}\right)^{7-x}$$

$$\left(-\frac{5}{6}\right)^{3/4}$$

$$\left(-\frac{5}{6}\right)^{7/6}$$

$$\left(-\frac{5}{6}\right)^{3/4 - 7/6} = \left(-\frac{5}{6}\right)^{7-x}$$

$$\left(-\frac{5}{6}\right)^{\frac{9-14}{12}} = \left(-\frac{5}{6}\right)^{7-n}$$

$$\left(-\frac{5}{6}\right)^{-\frac{3}{12}} = \left(-\frac{5}{6}\right)^{7-n}$$

$$7-n = -\frac{3}{4} \quad \text{or} \quad 28-4n = -3$$

$$\left(-\frac{5}{6}\right)^{-\frac{1}{4}} = \left(-\frac{5}{6}\right)^{7-n}$$

$$-\frac{1}{4} = 7-n$$

$$28-4n = -1, \quad 4n = 29$$

$$n = \frac{29}{4} = n = \frac{29 \times 3}{4 \times 3} = 87$$

(44)

$$\frac{\left(-\frac{5}{6}\right)^{3/4}}{\left(-\frac{5}{6}\right)^{7/6}} = \left(-\frac{5}{6}\right)^{7-n}$$

$$\left(-\frac{5}{6}\right)^{\frac{3}{4} - \frac{7}{6}} = \left(-\frac{5}{6}\right)^{7-n}$$

$$\left(-\frac{5}{6}\right)^{\frac{9-14}{12}} = \left(-\frac{5}{6}\right)^{7-n}$$

$$\left(-\frac{5}{6}\right)^{-\frac{3}{12}} = \left(-\frac{5}{6}\right)^{7-n}$$

$$7-n = -\frac{3}{12}$$

$$84-12n = -3$$

$$12n = 87$$

$$n = 87/12$$

(45)

$$4^n - 4^{n-1} = 24$$

$$4^n (1 - 4^{-1}) = 24,$$

$$4^n \left(1 - \frac{1}{4}\right) = 24, \quad 4^n \left(\frac{3}{4}\right) = 24,$$

$$4^n = \frac{24 \times 4}{3} = 32, \quad 4^n = 32$$

$$2^{2n} = 2^5, \quad 2n = 5, \quad (n = \frac{5}{2})$$

$$(2n)^n = \left(2 \times \frac{5}{2}\right)^{5/2} = 5^{5/2}$$

option (1)

(46)

$$\left(7^{-1/2} \times 5^2\right)^2 = \sqrt{25^3}$$

$$\frac{(7^{-1/2})^2 \times (5^2)^2}{((5^2)^3)^{1/2}} = \frac{7^{-1} \times 5^4}{5^3} = \frac{5}{7}$$

option (2)

(47)

$$(2d^2e^{-1})^3 \times \left(\frac{d^3}{e}\right)^{-2}$$

$$= 2^3 \cdot d^6 \cdot e^{-3} \times \frac{d^{-6}}{e^{-2}}$$

$$= 8 d^{6-6} e^{-3+2}$$

$$= 8 e^{-1} \quad \text{option (3)}$$

(48) $9^{1/2} \times 3^m = (27)^m$
 $(9^{1/2})^{1/2} \times 3^m = 3^{3m}$
 $3^1 \times 3^m = 3^{3m}$, $3^{1+m} = 3^{3m}$
 $1+m = 3m$, $1 = 3m - m$, $1 = 2m$, $m = 1/2$
 Option 4

(49) $\sqrt{9x} = \sqrt[3]{9^2}$
 $9^{x/2} = 9^{2/3}$, $x/2 = 2/3$, $x = 4/3$
 Option 2

(50) $a = x + \frac{1}{x}$
 $a^3 = (x + \frac{1}{x})^3$
 $a^3 = x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x} (x + \frac{1}{x})$
 $a^3 = x^3 + \frac{1}{x^3} + 3(a)$
 $x^3 + \frac{1}{x^3} = a^3 - 3a$
 (1.e) $x^3 + x \cdot 3 = a^3 - 3a$
 Option 2

(51) $a = x^{1/3} + x^{-1/3}$ — (1)
 $a^3 = (x^{1/3} + x^{-1/3})^3$
 $= (x^{1/3})^3 + (x^{-1/3})^3 + 3 \cdot x^{1/3} \cdot x^{-1/3} (x^{1/3} + x^{-1/3})$

$$a^3 = x - x - 1 - 3a$$

$$a^3 + 3a = x - x - 1$$

option (1)

(52)

$$a+b+c=0$$

$$\therefore (a^3+b^3+c^3) = 3abc$$

$$(a^3+b^3+c^3)^2 = (3abc)^2 = 9a^2b^2c^2$$

option (3)

(53)

$$a^x = b^y = c^z = 1, abc = 1,$$

$$\text{let } a^x = b^y = c^z = k$$

$$a^x = k, \quad a = k^{1/x}$$

$$b^y = k, \quad b = k^{1/y}$$

$$c^z = k, \quad c = k^{1/z}$$

$$abc = k^{1/x} \cdot k^{1/y} \cdot k^{1/z}$$

$$\text{But } abc = 1$$

$$1 = k^{1/x + 1/y + 1/z}$$

$$k^0 = k^{1/x + 1/y + 1/z}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

$$\frac{xy + yz + zx}{xyz} = 0, \quad xy + yz + zx = 0$$

option (3)

(54) $a^{x-1} = bc, b^{y-1} = ac, c^{z-1} = ab.$

$a^{x-1} = bc, a^x \cdot a^{-1} = bc, \frac{a^x}{a} = bc.$

$a^x = abc$

$b^{y-1} = ac, b^y \cdot b^{-1} = ac, \frac{b^y}{b} = ac, b^y = abc$

$c^{z-1} = ab, c^z \cdot c^{-1} = ab, \frac{c^z}{c} = ab, c^z = abc$

(1e) $a^x = b^y = c^z = abc.$

Let $a^x = b^y = c^z = abc = k.$

$a^x = k \Rightarrow a = k^{1/x},$

$b^y = k, \Rightarrow b = k^{1/y}$

$c^z = k, \Rightarrow c = k^{1/z}.$

$abc = k$

$abc = k^{1/x + 1/y + 1/z}$

$k^1 = k^{1/x + 1/y + 1/z}$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

$\frac{xy + yz + zx}{xyz} = 1,$

(1e) $xy + yz + zx = xyz$

$xy + yz + zx - xyz = 0$

Option 3

(55) $(a^{m^h})^n = a^{m^n}$

$$a^{mn} = a^{m^n}$$

$$mn = m^n$$

$$n = \frac{m^n}{m^1} = m^{n-1}$$

$$n = m^{n-1} \Rightarrow m = n^{\frac{1}{n-1}}$$

option (1)

(56) $\left(\frac{q}{p}\right)^{1-2x} = \left(\frac{p}{q}\right)^{\frac{1}{2}}$

$$1-2x = \frac{1}{2}, \quad 1 + \frac{1}{2} = 2x$$

$$\frac{1}{2} = 2x$$

$$x = \frac{1}{2 \times 2} = \frac{1}{4}$$

$x = \frac{1}{4}$

$$\left(\frac{p}{q}\right)^{2x-1} = \left(\frac{p}{q}\right)^{\frac{1}{2}}$$

$$2x-1 = \frac{1}{2}, \quad 2x = \frac{1}{2} + 1 = \frac{3}{2}$$

$$2x = \frac{3}{2}, \quad x = \frac{3}{2 \times 2} = \frac{3}{4}$$

$x = \frac{3}{4}$ option (1)

(57)

$$\left\{ \frac{a^{q-b}}{\sqrt[q]{a^{pq-q^2}}} \times a^{2(b-a)} \right\}^n$$

$$= \left[\frac{a^{q-b}}{a^{\frac{pq-q^2}{q}}} \times a^{2(b-a)} \right]^n$$

$$= \left[\frac{a^{q-b}}{a^{p-q}} \times a^{2b-2a} \right]^n \quad \left(\frac{pq-q^2}{q} = \frac{pq}{q} - \frac{q^2}{q} \right)$$

$$= \left[a^{q-b-(p-q)} \times a^{2b-2a} \right]^n$$

$$= \left[a^{q-b-p+q} \times a^{2b-2a} \right]^n$$

$$= \left[a^{2q-2p+2p-2q} \right]^n = (a^0)^n = 1^n = 1$$

option (3)

(58)

$$a+b+c=0$$

$$a^2 \cdot b^{-1} c^{-1} \cdot a^{-1} b^2 c^{-1} \cdot a^{-1} b^{-1} c^2$$

$$= a^3 \cdot \cancel{a^2} \cdot \frac{b^2}{bc} \cdot \frac{b^2}{ac} \cdot \frac{c^2}{ab}$$

$$= a^3 \cdot \frac{a^3}{abc} \cdot \frac{b^3}{abc} \cdot \frac{c^3}{abc}$$

$$= \frac{a^3+b^3+c^3}{abc} \cdot \frac{3abc}{abc} \quad [a+b+c=0]$$

$$= \frac{3abc}{abc} = 3$$

option (4)

(59)

$$\text{let } a^x = b^y = c^z = abc = k$$

$$a^x = k, \quad a = k^{1/x}$$

$$b^y = k, \quad b = k^{1/y}$$

$$c^z = k, \quad c = k^{1/z}$$

$$abc = k^{1/x} \cdot k^{1/y} \cdot k^{1/z}$$

$$k = k^{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

Option 3

(60)

$$2^{3^{45}} = 2^{9^{32}}$$

Option 2

(61)

$$\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$$

$$= \frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}}$$

$$= \frac{1}{\frac{x^b + x^a}{x^b}} + \frac{1}{\frac{x^a + x^b}{x^a}}$$

$$= \frac{x^b}{x^a + x^b} + \frac{x^a}{x^a + x^b}$$

$$= \frac{x^a + x^b}{x^a + x^b} = 1 \quad \text{Option 3}$$

(62)

$$\frac{2 \cdot 3^{n+1} + 7 \cdot 3^{n-1}}{3^{n+2} - 2 \left(\frac{1}{3}\right)^{1-n}}$$

$$= \frac{2 \cdot 3^n \cdot 3^1 + 7 \cdot 3^n \cdot 3^{-1}}{3^n \cdot 3^2 - 2 \cdot (3^{-1})^{1-n}} = \frac{2 \cdot 3^n \cdot 3^1 + 7 \cdot 3^n \cdot 3^{-1}}{3^n \cdot 3^2 - 2 \cdot 3^{-1+n}}$$

$$= \frac{2 \cdot 3^n \cdot 3^1 + 7 \cdot 3^n \cdot 3^{-1}}{3^n \cdot 3^2 - 2 \cdot 3^{-1} \cdot 3^n}$$

$$= \frac{3^n (2 \cdot 3^1 + 7 \cdot 3^{-1})}{3^n (3^2 - 2 \cdot 3^{-1})} \quad (\text{taking } 3^n \text{ as common})$$

$$= \frac{(6 + 7/3)}{(9 - 2/3)} = \frac{(18+7)/3}{(27-2)/3} = \frac{(25/3)}{(25/3)} = 1$$

option (1)

(63)

$$\frac{(2^{n+1})^m \cdot (2^2)^n \cdot 2^n}{(2^{m+1})^n \cdot 2^{2m}} = 1, \quad \frac{2^{mn+m} \cdot 2^{2n} \cdot 2^n}{2^{mn+n} \cdot 2^{2m}} = 1$$

$$\frac{2^{mn} \cdot 2^m \cdot 2^{2n} \cdot 2^n}{2^{mn} \cdot 2^n \cdot 2^{2m}} = 1$$

$$m+2n-2m = 0$$

$$m+2n-2m = 0$$

$$m+2n-2m = 0$$

$m+2n=0$
 $m=2n$
 option (3)

option (4)

option (3)

(54)

$$abc = 1$$

$$\begin{aligned} & \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} \\ &= \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} \\ &= \frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}} \end{aligned}$$

Since $abc = 1$, $\frac{1}{b} = ac$

$$\frac{1}{c} = ab$$

$$c = \frac{1}{ab}$$

$$= \frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+ab} + \frac{1}{1+ab+\frac{1}{a}} \left[\begin{array}{l} c = \frac{1}{ab} \\ \frac{1}{c} = ab \end{array} \right]$$

$$= \frac{1}{\frac{b+ab+1}{b}} + \frac{1}{1+b+ab} + \frac{1}{\frac{ab+1+b}{ab}}$$

$$= \frac{b}{1+b+ab} + \frac{1}{1+b+ab} + \frac{ab}{1+a+ab}$$

$$= \frac{1+b+ab}{1+b+ab} \Rightarrow \text{Ans}$$

65

$$x^y = y^x \Rightarrow y = x^{y/x} \quad \text{--- (1)}$$

$$x^y = y^x$$

$$x = y^{x/y}$$

Dividing by y on both sides

$$\frac{x}{y} = \frac{y^{x/y}}{y} = \left(\frac{y^{x/y}}{y^{x/y}} \right)^{x/y}$$

$$\left(\frac{x}{y} \right)^{x/y} = \frac{(y^{x/y})^{x/y}}{y^{x/y}} = \frac{x^{x/y}}{x}$$

$$\left(\frac{x}{y} \right)^{x/y} = x^{x/y - 1} \quad \text{Ans}$$

Option (3)

66

$$\left(\frac{x^a}{x^b} \right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c} \right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a} \right)^{c^2+ac+a^2}$$

$$= \left(x^{a-b} \right)^{a^2+ab+b^2} \cdot \left(x^{b-c} \right)^{b^2+bc+c^2} \cdot \left(x^{c-a} \right)^{c^2+ac+a^2}$$

$$= x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ac+a^2)}$$

$$= x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = 1 \quad \text{option (1)}$$

(67)

$$\frac{3 \cdot 2^n - 4 \cdot 2^n}{2^n - 2^{n-1}} = \frac{2^n (3-4)}{2^n - 2^{n-1}} = \frac{2^n (-1)}{2^n (1 - \frac{1}{2})}$$

$$= \frac{-1}{\frac{1}{2}} = -2$$

option (4)

(68)

$$\frac{p + p^2 + p^3 + p^4 + p^5 + p^6 + p^7}{p^{-3} + p^{-4} + p^{-5} + p^{-6} + p^{-7} + p^{-8} + p^{-9}}$$

$$= \frac{p(1 + p + p^2 + p^3 + p^4 + p^5 + p^6)}{p^{-9} + p^{-8} + p^{-7} + p^{-6} + p^{-5} + p^{-4} + p^{-3}}$$

Since p^{-9} is the least power, therefore p^{-9} taken common from the denominator

$$= \frac{p(1 + p + p^2 + p^3 + p^4 + p^5 + p^6)}{p^{-9}(1 + p + p^2 + p^3 + p^4 + p^5 + p^6)}$$

$$= \frac{p^1}{p^{-9}} = p^{1+9} = p^{10}$$

option (1)

69) $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

$$= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{\cancel{a^2-b^2} + \cancel{b^2-c^2} + \cancel{c^2-a^2}}$$

$$= x^0$$

$$= 1 \quad \text{option (2)}$$

70) $\left(\frac{64}{125}\right)^{-2/3} \div \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$

$$\left(\frac{4^3}{5^3}\right)^{-2/3} \div \frac{1}{\left(\frac{4^4}{5^4}\right)^{1/4}} + 1 \quad [\because x^0 = 1]$$

$$\left(\frac{4}{5}\right)^{-2} \div \frac{1}{\left(\frac{4}{5}\right)^{1/4}} + 1$$

$$\left(\frac{4}{5}\right)^{-2} \div \frac{1}{\frac{4}{5}} + 1$$

$$\left(\frac{5}{4}\right)^2 \div \frac{5}{4} + 1$$

$$\frac{25}{16} \div \frac{5}{4} + 1$$

$$\frac{25}{16} \times \frac{4}{5} + 1$$

$$\frac{25}{4} + 1$$

$$\frac{25}{4} + \frac{4}{4} = \frac{29}{4}$$

$$\begin{aligned} & \left(\frac{5}{4}\right)^2 \times \frac{4}{5} + 1 \\ &= \frac{25}{16} \times \frac{4}{5} + 1 \\ &= \frac{5}{4} + 1 = \frac{9}{4} \quad \text{option (2)} \end{aligned}$$

$$(71) \quad \frac{4^{n+1} \cdot 2^n - 8^n}{2^{2n}} = \frac{3}{8}$$

$$\frac{4^n \cdot 4 \cdot 2^n - 2^{3n}}{2^{2n}} = \frac{3}{8}$$

$$\frac{2^{2n} \cdot 2^n \cdot 4 - 2^{3n}}{2^{2n}} = \frac{3}{8}$$

$$\frac{2^{3n} (4-1)}{2^{2n}} = \frac{3}{8} \quad , \quad \frac{2^{3n} \times 3}{2^{2n}} = \frac{3}{8}$$

$$2^{3n-2n} = \frac{1}{2^3} = 2^{-3}$$

$$2^n = 2^{-3} \Rightarrow n = -3$$

option (3)

(72)

$$(72) \quad \frac{\left(p^2 - \frac{1}{q^2}\right)^p \cdot \left(p - \frac{1}{q}\right)^{q-p}}{\left(q^2 - \frac{1}{p^2}\right)^q \cdot \left(q + \frac{1}{p}\right)^{p-q}}$$

$$= \frac{\left\{\left(p - \frac{1}{q}\right)\left(p + \frac{1}{q}\right)\right\}^p \cdot \left(p - \frac{1}{q}\right)^q \cdot \left(p - \frac{1}{q}\right)^{-p}}{\left\{\left(q - \frac{1}{p}\right)\left(q + \frac{1}{p}\right)\right\}^q \cdot \left(q + \frac{1}{p}\right)^p \cdot \left(q + \frac{1}{p}\right)^{-q}}$$

$$= \frac{\cancel{\left(p - \frac{1}{q}\right)^p} \cdot \left(p + \frac{1}{q}\right)^p \cdot \left(p - \frac{1}{q}\right)^q \cdot \cancel{\left(q + \frac{1}{p}\right)^q}}{\left(q - \frac{1}{p}\right)^q \cdot \cancel{\left(q + \frac{1}{p}\right)^q} \cdot \left(q + \frac{1}{p}\right)^p \cdot \cancel{\left(p - \frac{1}{q}\right)^p}}$$

$$= \frac{\left(p + \frac{1}{q}\right)^p}{\left(q + \frac{1}{p}\right)^p} \times \frac{\left(p - \frac{1}{q}\right)^q}{\left(q - \frac{1}{p}\right)^q}$$

$$= \left\{ \frac{\frac{pq+1}{q}}{q} \right\}^p \times \left\{ \frac{\frac{pq+1}{q}}{q} \right\}^q$$

$$= \left(\frac{p}{q}\right)^p \left(\frac{p}{q}\right)^q = \left(\frac{p}{q}\right)^{p+q}$$

option 3

(73)

$$\begin{aligned}
 & \frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{c-a}+x^{b-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} \\
 &= \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}} + \frac{1}{1+\frac{x^c}{x^a}+\frac{x^b}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} \\
 &= \frac{x^c+x^b+x^a}{x^c} + \frac{x^a+x^c+x^b}{x^a} + \frac{x^b+x^a+x^c}{x^b} \\
 &= \frac{x^c}{x^a+x^b+x^c} + \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^a+x^b+x^c} \\
 &= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1
 \end{aligned}$$

option: (1)

(74)

$$\begin{aligned}
 & \left(\frac{x^m}{x^n} \right)^{m^2+mn+n^2} \times \left(\frac{x^n}{x^l} \right)^{n^2+nl+l^2} \times \left(\frac{x^l}{x^m} \right)^{l^2+lm+m^2} \\
 &= \left(x^{m-n} \right)^{m^2+mn+n^2} \times \left(x^{n-l} \right)^{n^2+nl+l^2} \times \left(x^{l-m} \right)^{l^2+lm+m^2} \\
 &= x^{m^3-n^3} \times x^{n^3-l^3} \times x^{l^3-m^3} \\
 &= x^{\cancel{m^3-n^3} + \cancel{n^3-l^3} + \cancel{l^3-m^3}} = x^0 = 1
 \end{aligned}$$

option: (1)

(75) $pqr = 1$

$$\frac{1}{1+p+q-1} + \frac{1}{1+q+r-1} + \frac{1}{1+r+p-1}$$

$$= \frac{1}{1+p+q} + \frac{1}{1+q+\frac{1}{p}} + \frac{1}{1+r+\frac{1}{p}}$$

$pqr = 1$, ~~$px = \frac{1}{q}$~~

$\boxed{pq = \frac{1}{r}}$

$\boxed{r = \frac{1}{pq}}$

$$= \frac{1}{1+p+\frac{1}{q}} + \frac{1}{1+q+pq} + \frac{1}{1+\frac{1}{pq}+p}$$

$$= \frac{1}{\frac{q+pq+1}{q}} + \frac{1}{1+q+pq} + \frac{1}{\frac{1+pq+p}{pq}}$$

$$= \frac{q}{1+pq+q} + \frac{1}{1+q+pq} + \frac{pq}{1+p+pq}$$

$$= \frac{1+q+pq}{1+q+pq} = 1$$

option (1)